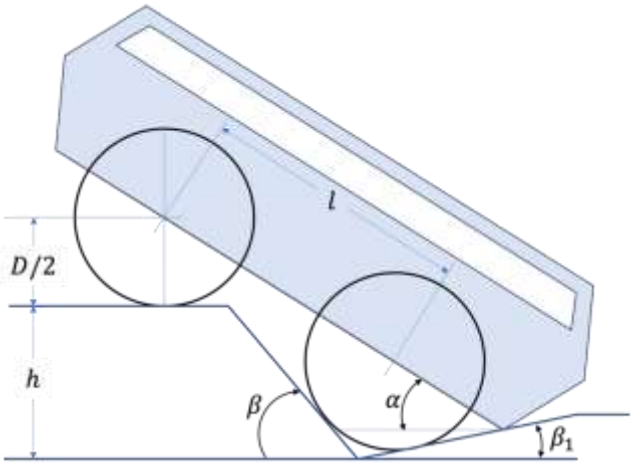
Numerical Methods, spring 2025
Homework 2

To design the Mars rover, ESA engineers must account for two common failure modes when navigating rough terrain. The first occurs when the vehicle's underside makes contact with the ground, and the second happens when the vehicle descends into a ditch, causing its nose to touch the surface (see below).



The mathematical model of the rover indicates that the maximum angle α it can traverse, given the maximum angle β at which its underside remains clear of the ground, satisfies the following equation:

$$f(\alpha) = F\,\sin(\alpha)\cos(\alpha) + G\sin^2(\alpha) - H\cos(\alpha) - E\sin(\alpha) = 0$$

where $F = l \cdot \sin(\beta_1)$, $G = l \cdot \cos(\beta_1)$,$H = \left(h + \frac{D}{2}\right) \cdot \sin(\beta_1) - \frac{D}{2} \cdot \tan(\beta_1)$, and $E = \left(h + \frac{D}{2}\right) \cdot \cos(\beta_1) - \frac{D}{2}$.

a) For $l = 2.26\ m$, $h = 1.25\ m$, $D = 1.40\ m$, and $\beta_1 = 11.5°$,
   a. Plot the function $f(\alpha)$;
   b. Solve the equation with the python built-in scipy.optimize.root function with the tolerance *tol = 1e-12*. This solution should be considered as the true solution when computing the error in part b)

b) Compute the angle α with the accuracy of $0.001°$ using your written code implementing (i) Regula Falsi, (ii) Secant, and (iii) Newton-Raphson method. How many iterations did you need for each case? Print out a table for each case in the format "Iteration i: $f(\alpha_i) = \cdots, \alpha_i = \cdots, \text{error} = \cdots$. Plot the evolution of error in three cases and discuss the obtained results.

c) Repeat the calculation from part b) for $D = 0.76\ m$. Discuss the results and compare them to the previous ones.